Overview

Broadly speaking, questions in extremal combinatorics ask how large or small a combinatorial object can be. Extremal combinatorics utilizes many tools from other areas of mathematics, and it has a particularly strong connection with probability. For example, Erdos used random graphs to give the first exponential lower bound for diagonal Ramsey numbers, which was a major open problem.

Ever since this seminal result of Erdos, extremal combinatorics has become deeply intertwined with probability, resulting in what is commonly referred to as the probabilistic method. Indeed, many of the recent advances in extremal combinatorics have utilized random variables, concentration inequalities, as well as many other tools and ideas originating from probability.

A recent trend is to not only study random objects as a tool for solving extremal combinatorics problems, but also as a source of extremal problems to study. One classical problem in this direction is to determine the maximum (expected) number of edges in a triangle-free subgraph of a random graph. We consider generalizations of this problem by looking for large F-free subgraphs of random graphs and hypergraphs. In a similar spirit, this proposal considers problems related to maximum and minimum scores in a certain card guessing game involving a randomly shuffled deck of cards.

Intellectual Merit

One of the central problems in extremal combinatorics is to determine the maximum number of edges that a graph can have if it avoids a given subgraph. This proposal considers a random generalization of this problem by investigating the (expected) maximum number of edges in a subgraph of a random graph that avoids a given subgraph; and more generally the proposal considers the analogous problem for random hypergraphs. The PI together with Verstraete has proven a number of results in this direction when forbidding loose cycles, Berge cycles, or complete r-partite r-graphs. The PI plans to refine these bounds for hypergraph cycles, as well as to extend these results to other forbidden subgraphs.

Motivated by real-world problems related to clinical trials and casino games, Diaconis and Graham studied the "partial feedback model", which is a one player game involving a uniformly shuffled deck of cards. The PI together with Diaconis, Graham, and He answered a 40 year old problem of Diaconis and Graham, determining the maximum number of points a player can achieve in this game. This problem was particularly difficult to solve because the optimal strategy for this game is unknown, and careful probabilistic arguments were needed to overcome this obstacle. Many related problems remain, such as determining the minimum expected score that can be achieved, as well as investigating this game when the deck is shuffled "adversarially" as opposed to uniformly at random.

Broader Impact

The card guessing game considered in this proposal was motivated by real world problems, and it is likely that further work on these problems could lead to additional applications. The PI has and will continue to discuss their work at conferences, seminars, and by writing expository notes related to their research.

The PI has helped bolster the mathematical community at large by refereeing over 20 research articles, by leading a research project for the Graduate Research Workshop in Combinatorics, and by co-organizing the Graduate Student Combinatorics Conference 2022.

At UC San Diego (UCSD), the PI co-organized two seminars, two reading courses, and a number of social events for graduate students. The PI has served as a mentor for undergraduate students through UCSD's Association for Women in Mathematics chapter, as well as for several first year graduate students. The PI is an active member of UCSD's Mathematics Graduate Student Council, and in particular helped create a bootcamp for incoming graduate students.